MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

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WILLIAM LASSELL, Esq., President, in the Chair.

Lieut. Abney, Royal Engineers,
R. Abbay, Esq. Wadham College,
W. Thirlwall Bayne, Esq., 34 Jermyn Street,
C. J. Lambert, Esq., Pembroke College, Cambridge,
Dr. Riches, 1 Lansdowne Crescent, Bath, and
Geo. M. Seabroke, Esq., Rugby,

were balloted for and duly elected Fellows of the Society.

The following Circular has been, by direction of the Council, issued to the Fellows of the Society:—

Sir,—We are instructed to communicate to you the following resolution, which was passed at a Committee of the Council held yesterday, April 8th,

Resolved—"That the Fellows be informed that there is a possibility of the Government providing means of transit to and from stations on the Mediterranean for about sixty observers, who may be willing to take part in the Observation of the Total Eclipse of December 22, 1870; and that persons willing to undertake a portion of the Observation, on a plan to be arranged by the Council, be invited to send their names to the Secretaries, and also to state the branch of observation which they would be prepared, or prefer, to undertake, and the instruments they would be willing to contribute."

It is desirable that the names of those who are willing to take part in the observation of the Eclipse should be sent in, if possible, before the next Meeting of Council, May 13.

We have the honour to remain, Sir, yours faithfully,

WILLIAM HUGGINS,
E. J. STONE, } *Hon. Secs.*

Royal Astronomical Society, April 9, 1870.

On the Orbit of the Comet of 1683. By Mr. W. E. Plummer, of Mr. Bishop's Observatory, Twickenham. (Note by Mr. Hind.)

The comet of 1683 was first seen by Flamsteed on the 23rd of July, and was assiduously observed at Greenwich until the 5th of September. The original observations are printed in the first volume of the *Historia Cœlestis*. Hevelius observed the comet at Dantzic from the 30th of July to the 4th of September, and the particulars of his observations appear in his *Annus Climactericus*.

Halley calculated a parabolic orbit for this comet from Flamsteed's observations, which was published in his *Synopsis of Cometary Astronomy*, and is as follows :—

Perihelion Passage, 1683, July 13, at $3^h\ 0^m$, G.M.T.

Longitude of Perihelion	85° 29′ 30″
Longitude of Ascending Node ..	173 23 0
Inclination of the Orbit	83 11 0
Logarithm of Perihelion distance ..	9·748343

Motion Retrograde.

In the fifth volume of the *Astronomische Nachrichten* (No. 117) Clausen (the present director of the Observatory at Dorpat) has given elliptic elements for this comet, founded as he states upon a new reduction of Flamsteed's observations. These elements are as follow :—

Perihelion Passage, 1683, July 12·73236, M.T. at Paris.

Longitude of Perihelion	86° 31′ 15″	Mean Equinox, 1683·0.
Longitude of Ascending Node ..	173 17 48	
Inclination of the Orbit	83 47 46	
Logarithm of Perihelion distance ..	9·7430148	
Eccentricity	0·9832470	

Motion Retrograde.

The period of revolution in this ellipse is 189·8 years, which would bring the comet to perihelion again in 1873.

Mr. Bishop having, on my suggestion, requested Mr. Plummer, assistant in his observatory, to make a new and rigorous reduction of Flamsteed's observations, an accurate Ephemeris of the comet was first calculated for the whole interval between the extreme observations, the Sun's longitudes and the radii-vectores of the Earth, being deduced from the Solar Tables of Leverrier. The places of the comparison stars were carried back, from the Greenwich Catalogue for 1860, undoubtedly the highest existing

authority. The observed distances of the comet from stars were carefully corrected for refraction and parallax, a mean value of the refraction being employed, and the distances in the computation of parallax inferred from Clausen's ellipse. A comparison of the observations thus rigorously reduced with Clausen's orbit was then made, and it was at once evident that an orbit might be found to represent the observations much more satisfactorily than the ellipse in question. This Mr. Plummer proceeded to investigate, and the result is a parabolic orbit which accords as well with the whole course of observations as the limits of their errors will allow. The elements of this parabola are :—

Perihelion passage, 1683, July 13, 09068, Greenwich M.T.

Longitude of Perihelion	85° 35′ 59″·5	Mean Equinox, 1683·0.
Longitude of Ascending Node ..	173 24 39·7	
Inclination of the Orbit	83 13 14·7	
Logarithm of Perihelion distance ..	9·7478656	

Motion Retrograde.

The sums of the squares of the errors are :—

	R.A.	N.P.D.
In Clausen's ellipse ..	337278″	224845″
In Plummer's parabola ..	29089″	58394″

It would therefore appear that there is no reason to expect the return of the comet within the next few years, or to suppose that it is one of moderate period.

If I am not greatly mistaken, Clausen has merely reduced Flamsteed's observations on three days, and computed an orbit upon them by a general method, such as that given by Gauss in the *Theoria Motus*. The observations selected are almost indicated by the comparison with his elements.

I should add that the observations of Hevelius were likewise rigorously reduced, but being found greatly inferior to Flamsteed's in point of accuracy, it was considered that no advantage would be gained by employing them in the calculations for the orbit.

It is the intention of Mr. Bishop, to publish the details of Mr. Plummer's investigation in a separate form.

1870, *April* 7.

Studies on the frequency of Sun-spots, and on their connexion with the Magnetic Declination-variation. By Prof. Rudolf Wolf. (Translation.)

Denoting by r the Relative-number for the Sun-spot frequency introduced by me in the year 1850, by f the number of days with-

out spots, and by b the number of days of observation, I obtain for 1864–69 the following table:—

	1864.		1865.		1866.	
	r	$f:b$	r	$f:b$	r	$f:b$
January	57·5	0 : 31	48·3	0 : 30	33·3	0 : 31
February	47·2	0 : 29	44·8	0 : 27	39·4	1 : 28
March	67·3	0 : 31	40·7	2 : 28	27·2	0 : 29
April	30·0	2 : 29	32·5	1 : 30	18·9	2 : 30
May	40·9	0 : 31	37·5	2 : 31	15·0	6 : 31
June	58·3	0 : 30	36·3	2 : 30	18·3	3 : 30
July	57·2	0 : 31	29·7	2 : 31	10·2	9 : 31
August	57·9	2 : 31	40·3	0 : 31	14·0	5 : 31
September	30·5	1 : 30	22·9	7 : 30	8·0	13 : 30
October	35·5	0 : 31	18·5	10 : 31	14·6	5 : 31
November	59·1	0 : 28	24·7	4 : 27	9·3	16 : 30
December	24·1	1 : 24	13·3	9 : 28	1·6	26 : 30
Year	47·1	6 : 356	32·5	39 : 354	17·5	86 : 362

	1867.		1868.		1869.	
	r	$f:b$	r	$f:b$	r	$f:b$
January	0·0	29 : 29	12·2	13 : 25	72·4	0 : 27
February	0·8	26 : 28	16·4	5 : 28	72·4	1 : 24
March	10·8	11 : 31	28·7	3 : 30	65·3	0 : 28
April	5·8	20 : 30	39·4	0 : 30	46·5	1 : 30
May	3·3	24 : 31	30·3	3 : 31	115·8	0 : 30
June	1·6	26 : 30	34·7	2 : 30	120·4	0 : 30
July	5·3	18 : 31	32·2	10 : 31	65·1	1 : 31
August	5·9	19 : 31	38·6	0 : 31	93·2	0 : 30
September	10·6	15 : 30	52·6	1 : 30	88·5	0 : 30
October	14·2	13 : 31	60·5	0 : 29	62·4	0 : 30
November	10·3	9 : 30	67·9	0 : 21	85·7	0 : 24
December	27·5	6 : 24	68·4	0 : 29	122·1	0 : 25
Year	8·0	216 : 356	40·2	32 : 345	84·1	3 : 339

One recognises herein, at the first glance, the minimum of 1867 according with my Sun-spot period of $11\frac{1}{9}$ years, and from that date the rapidly increasing Sun-spot frequency. By the empirical formula which, since 1859, I proposed for various points in order from r to deduce the declination-variation b, I give here, only by way of example, the calculated formula 1852–61 for Christiania,—

$$v = 0'{\cdot}0413\, r + 4'{\cdot}921$$

This gives the following table, where v denotes the variations calculated by the formula, v' the values resulting from the actual

observations, communicated to me by Messrs. Mohn and Fearnley.

	1864.	1865.	1866.	1867.	1868.	1869.
v	6′·87	6′·26	5′·64	5′·25	6′·58	8′·39
v'	6·00	5·72	5·70	5·69	6·65	7·82

It is hardly necessary to remark that the constants of the formulæ may, by means of the whole series of the Christiania Observations, easily be altered so as to obtain a yet better agreement. I prefer, however, for the present not to make this change.

Zurich, March 28, 1870.

Observations of Lunar Eclipse, Jan. 17, 1870.

By J. Tebbutt, jun.

The lunar eclipse of the 17th instant was remarkably well seen here. The Moon was overspread with very thin filmy cloud till about $11^h\ 43^m$, but the diminution of her brilliancy from that cause was very slight. She remained unclouded during the rest of the phenomenon. No decided defalcation of light was noticed on the eastern limb till $10^h\ 41^m$, but at $10^h\ 52^m$ the effects of the penumbra were very marked. The following are the local mean times of the different phases as near as they could be observed, it being a most difficult matter to fix the precise instants of the contacts owing to the ill-defined character of the shadow:—

	d	h	m	s
First contact with the shadow	17	11	1	19
Beginning of the total phase	..	12	0	29
End of the total phase	..	13	38	53
Last contact with the shadow	..	14	38	58

At $11^h\ 29^m$ the shadow assumed a light copper tint, except at its periphery, where it was of a very dark green. The copper tint, as seen in the telescope, appeared to extend even to the filmy cloud which lay along the Moon's eastern limb. At $11^h\ 43^m$, when the Moon shone unclouded, the details on the obscured portion of the lunar surface began to be perceptible in the telescope. These became gradually more distinct, and it was soon observed that the dark body of the Moon was surrounded by numerous telescopic stars, and that many occultations would occur during the total phase. Several of these phenomena were observed with tolerable accuracy; some of the stars, however, were too faint for accurate observation. The following occultations were recorded:—

Star.	Mag.	Phase.	Mean Time. h	m	s	Remarks.
a	7	Disappearance	11	59	24·7	Disappearance sudden.
b	8½	,,	12	49·4		(Approx.)
c	8	,,	13	2	19·3	Disappearances near upper limb : a little uncertain.
d	8	,,	13	5	0·2	
e	8	Reappearance	13	6	18·7	
a	7	,,	13	25	4·2	(Approx.)
f	7	Disappearance	13	36	24·2	Uncertain to a second, owing to increasing brightness of limb.

The noted time of the reappearance of the star *a*, owing to a temporary removal of the eye from the telescope, was probably two or three seconds late. The Moon's disk was of a copper hue throughout the total phase, and continued distinctly visible both to the naked eye and in the telescope. The southern limb was remarkably bright at the middle of the eclipse. The meridian transit of the first limb was pretty well observed, but the second limb was too faint. The copper and dark green tints were again observed after the total phase, that portion of the obscured surface next to the centre of the shadow being copper-tinted, and the outline of the shadow being very dark green. The telescopic observations during the eclipse were all made with my refractor of 3¼ inches aperture, and 48 inches focal length, furnished with a magnifying power of about 30.

Windsor, New South Wales, Jan. 26, 1870.

A few further Notes on the Floor of Plato.

By W. R. Birt, Esq.

In my communication on the floor of *Plato*, which was read at the Meeting of the Society in November last, I solicited attention to a feature presented by the spots, and which I ventured to term "degree of visibility." The paper had reference to twenty-five spots, the degree of visibility of sixteen being given. The entire number of observations to Sept. 27, 1869, was 238. Since that date the observations have been continued, and greater attention has been given to the subject, and as twelve months' observations are now completed, including a luni-solar year, the following results may not be uninteresting.

The number of spots which have been seen on the floor of *Plato* up to the present time is thirty-five, eight of which have been detected since Sept. 27, 1869. The number of observations since that date amounts to 531, being more than double the number (240) during the first six lunations of the year 1869, April to 1870 March inclusive. In the following table the "degree of visibility" of each spot is given for the first six lunations, the last six lunations, the increase or decrease of visibility of

those spots which are comparable, and the "degree of visibility" of each spot for the year, the number of observations being 771.

No.	April to Sept. Obs.	Vis.	Oct. to March. Obs.	Vis.	Diff.	Year. Obs.	Vis.
0	..	..	5	·070	..	5	·048
1	33	1·000	71	1·000	..	104	1·000
2	1	·030	6	·084	+·054	7	·067
3	24	·727	68	·958	+·231	92	·885
4	28	·848	62	·873	+·025	90	·865
5	20	·606	30	·423	−·183	50	·481
6	8	·242	24	·338	+·096	32	·308
7	8	·242	8	·113	−·129	16	·154
8	..	..	3	·042	..	3	·029
9	5	·151	20	·282	+·131	25	·240
10	6	·182	5	·070	−·112	11	·106
11	1	·030	14	·197	+·167	15	·144
12	3	·091	1	·014	−·077	4	·038
13	16	·485	12	·169	−·316	28	·269
14	20	·606	30	·423	−·183	50	·481
15	1	·030	2	·028	−·002	3	·029
16	12	·364	20	·282	−·082	32	·308
17	22	·666	62	·873	+·207	84	·808
18	1	·030	6	·084	+·054	7	·067
19	17	·515	10	·141	−·374	27	·260
20	3	·091	5	·070	−·021	8	·077
21	..	..	4	·056	..	4	·038
22	10	·303	15	·211	−·092	25	·240
23	1	·030	3	·042	+·012	4	·038
24	..	..	5	·070	..	5	·045
25	..	..	9	·127	..	9	·085
26	..	..	1	·014	..	1	·010
27	..	..	2	·028	..	2	·019
28	..	..	1	·014	..	1	·010
29	..	..	6	·084	..	6	·058
30	..	..	8	·113	..	8	·077
31	..	..	1	·014	..	1	·010
32	..	..	6	·084	..	6	·058
33	..	..	1	·014	..	1	·010
34	..	..	5	·070	..	5	·048

On running the eye along the column of differences it will be seen that the number of spots in which an increase has taken place during the last six lunations is nearly equal to that in which a decrease has occurred, viz. ten of the former and eleven of the latter. Spot No. 3, a craterlet, has manifested the greatest increase, while spot No. 19 has exhibited the largest decrease.

The extent of variation of the separate spots is very irregular, and does not appear to indicate the operation of any general law. In one or two instances only have neighbouring spots been similarly affected ; thus spots Nos. 5 and 14 in the S.W. quadrant of *Plato*, exhibit the same *decrease* of visibility, and the way in which they have varied from lunation to lunation is somewhat similar, and unlike the manner in which most of the other spots have varied. Spots No. 2 and 18 exhibit the same *increase* of visibility. The great increase of white spots in every part of the Moon's disk, about the time of full, dependent upon the value of $\mathbb{C} - \odot$ would, lunation after lunation, contribute to a steady value of the degree of visibility rather than the irregularity which is indicated by the observations if the *same* spots had been seen month after month. Although the observations amount to 771, and as many as twenty-two spots have been observed on one evening, the average number visible at any given time, as deduced from the 108 series of observations, is seven only, a number which is constant. Upon examining those series in which a smaller number than seven has been recorded, it is found that, besides the spots most commonly seen, viz. Nos. 1, 3, 4, and 17, the remaining two have not been the same. The additional spots seen on these occasions have been very various, several of them having low degrees of visibility, and some of these, which it might be expected could be seen only in the finest weather, have been observed in ordinary states of the Earth's atmosphere.

The observations of the twelve lunations ending in March, 1870, extend considerably the basis on which to found an intelligible explanation of the phenomena, it is, nevertheless, much too narrow to hazard more than conjecture. Another year's observations will doubtless throw further light on the subject.

On the Graphical Construction of the Umbral or Penumbral Curve at any instant during a Solar Eclipse. By Prof. Cayley.

The curve in question, say the penumbral curve, is the intersection of a sphere by a right cone,—I wish to show that the stereographic projection of this curve may be constructed as the envelope of a variable circle, having its centre on a given conic, and cutting at right angles a fixed circle ; this fixed circle being in fact the projection of the circle which is the section of the sphere by the plane through the centre and the axis of the cone, or say by the axial plane. The construction thus arrived at is Mr. Casey's construction for a bicircular quartic ; and it would not be difficult to show that the stereographic projection of the penumbral curve is in fact a bicircular quartic.

The construction depends on the remark that a right cone is the envelope of a variable sphere having its centre on a given line, and its radius proportional to the distance of the centre

from a given point on this line; and on the following theorem of plane geometry:

Imagine a fixed circle, and a variable circle having its centre on a given line, and its radius proportional to the distance of the centre from a given point on the line (or, what is the same thing, the variable circle always touches a given line); then the locus of the pole in regard to the fixed circle, of the common chord of the two circles (or, what is the same thing, the locus of the centre of a new variable circle which cuts the fixed circle at right angles in the points where it is met by the first-mentioned variable circle) is a conic.

To fix the ideas, say that P is the centre of the first variable circle; A B its common chord with the fixed circle; Q the centre of the circle which cuts the fixed circle at right angles in the points A and B; then the locus of Q is a conic.

To prove this, take $x^2 + y^2 = 1$ for the equation of the fixed circle $(x - \alpha)^2 + (y - \beta)^2 = \gamma^2$ for that of the variable circle; the foregoing law of variation being in fact such that α, β, γ, are linear functions of a variable parameter θ; the equation of the common chord A B is $-2\alpha x - 2\beta y + 1 + \alpha^2 + \beta^2 - \gamma^2 = 0$; viz., this equation contains θ quadratically; hence the envelope of the common chord is a conic; and thence (reciprocating in regard to the fixed circle) the locus of the pole of A B, that is, of the point Q, is also a conic.

Consider now a solid figure in which the circles are replaced by spheres; viz. we have a fixed sphere, and a variable sphere having its centre on a given line and its radius proportional to the distance of the centre from a given point on the line. The envelope of the variable sphere is a right cone; the intersection of the cone with the fixed sphere is the envelope of the small circle of the sphere, say the circle A B, which is the intersection of the fixed sphere by the variable sphere. This circle A B is also the intersection of the fixed sphere by a sphere, centre Q, which cuts the fixed sphere at right angles; and by what precedes the locus of Q is a conic. Hence the penumbral curve is given as the envelope of the circle A B which is the intersection of the fixed sphere by a sphere which has its centre Q on a conic, and which cuts the fixed sphere at right angles. It is obvious that the circle A B always cuts at right angles the great circle which is the section of the fixed sphere by the axial plane, or say the axial circle. Project the whole figure stereographically; the projection of the circle A B is a variable circle which cuts at right angles the circle which is the projection of the axial circle, and which has for its centre the point Q′ which is the projection of Q. But the locus of Q being a conic, the locus of its projection Q′ is also a conic; and we have thus the projection of the penumbral curve as the envelope of a variable circle which has its centre on a conic, and which cuts at right angles a fixed circle.

We may in the axial plane construct five points of the conic which is the locus of Q, by means of any five assumed positions

of the variable circle, and somewhat simplify the construction by a proper choice of the five positions of the variable circle. This is not a convenient construction, and even if it were accomplished we should still have to construct the projection of the conic so obtained, in order to find, in the figure of the stereographic projection, the conic which is the locus of Q′. I do not at present perceive any direct construction for the last-mentioned conic; but assuming that a tolerably simple construction can be obtained, the construction of the projection of the penumbral curve as the envelope of the variable circle is as easy and rapid as possible. Probably the easiest course would be (without using the conic at all) to calculate numerically, for a given position of the variable sphere, the terrestrial latitude and longitude of the two points of intersection of the variable sphere by the axial circle; laying these down on the projection, we have then a position of the variable circle; and a small number of properly selected positions would give the penumbral curve with tolerable accuracy.

I have throughout spoken of the penumbral curve, as it is in regard hereto that a graphical construction is most needed; but the theory is applicable, without any alteration, to the umbral curve.

On the Geometrical Theory of Solar Eclipses.

By Prof. Cayley.

The fundamental equation in a solar eclipse is, I think, most readily established as follows:—

Take the centre of the Earth for origin, and consider a set of axes fixed in the Earth and moveable with it; viz., the axis of z directed towards the North Pole; those of x, y, in the plane of the Equator; the axis of x directed towards the point longitude 0°; that of y towards the point longitude 90° W. of Greenwich. Take a, b, c, for the co-ordinates of the Moon; k for its radius (assuming it to be spherical); a', b', c', for the co-ordinates of the Sun; k' for its radius (assuming it to be spherical); then, writing $\theta + \varphi = 1$, the equation

$$\{\theta(x-a) + \varphi(x-a')\}^2 + \{\theta(y-b) + \varphi(y-b')\}^2 + \{\theta(z-c) + \varphi(z-c')\}^2 = (\theta k \pm \varphi k')^2$$

is the equation of the surface of the Sun or Moon, according as $\theta, \varphi = 1, 0$ or $= 0, 1$: and for any values whatever of θ, φ, it is that of a variable sphere, such that the whole series of spheres have a common tangent cone. Writing the equation in the form

$$\begin{aligned} &\theta^2\{(x-a)^2 + (y-b)^2 + (z-c)^2 - k^2\} \\ &+ 2\theta\varphi\{(x-a)(x-a') + (y-b)(y-b') + (z-c)(z-c') \mp kk'\} \\ &+ \varphi^2\{(x-a')^2 + (y-b')^2 + (z-c')^2 - k'^2\} = 0, \end{aligned}$$

or, putting for shortness,

$$\begin{aligned}
\varrho &= a^2 + b^2 + c^2 - k^2 \\
\varrho' &= a'^2 + b'^2 + c'^2 - k'^2 \\
\sigma &= a\,a' + b\,b' + c\,c' \mp k\,k' \\
\mathrm{P} &= a\,x + b\,y + c\,z \\
\mathrm{P}' &= a'\,x + b\,y + c'\,z,
\end{aligned}$$

the equation is

$$\begin{aligned}
&\theta^2\ (x^2 + y^2 + z^2 - 2\,\mathrm{P} + \varrho) \\
+\ &2\,\theta\varphi\ (x^2 + y^2 + z^2 - \mathrm{P} - \mathrm{P}' + \sigma) \\
+\ &\varphi^2\ (x^2 + y^2 + z^2 - 2\,\mathrm{P}' + \varrho') = 0
\end{aligned}$$

and the equation of the envelope consequently is

$$(x^2 + y^2 + z^2 - 2\,\mathrm{P} + \varrho)\ (x^2 + y^2 + z^2 - 2\,\mathrm{P}' + \varrho') - x^2 + y^2 + z^2 - \mathrm{P} - \mathrm{P}' + \sigma)^2 = 0$$

that is

$$(x^2 + y^2 + z^2)\ (\varrho + \varrho' - 2\,\sigma) - (\mathrm{P} - \mathrm{P}')^2 - 2\,(\varrho' - \sigma)\,\mathrm{P} - 2\,(\varrho - \sigma)\,\mathrm{P}' + \varrho\,\varrho' - \sigma^2 = 0$$

which is the equation of the cone in question.

Observe that one sphere of the series is a *point*, viz., taking first the upper signs, if we have $\theta\,k + \varphi\,k' = 0$, that is

$$\theta = \frac{k'}{k' - k},\quad \varphi = \frac{-k}{k' - k},$$

then the sphere in question is the point the co-ordinates whereof are

$$x = \frac{k'\,a - k\,a'}{k' - k},\quad y = \frac{k'\,b - k\,b'}{k' - k},\quad z = \frac{k'\,c - k\,c'}{k' - k}$$

which point is the vertex of the cone: it hence appears that, taking the upper signs, the cone is the *umbral* cone, having its vertex on this side of the Moon; and similarly taking the lower signs, then if we have $\theta\,k - \varphi\,k' = 0$, that is

$$\theta = \frac{k'}{k' + k},\quad \varphi = \frac{k}{k' + k},$$

then the variable sphere will be the point the co-ordinates of which are

$$\frac{k'\,a + k\,a'}{k' + k},\qquad \frac{k'\,b + k\,b'}{k' + k},\qquad \frac{k\,c' + k'\,c}{k' + k},$$

which point is the vertex of the cone; viz. the cone is here, the penumbral cone having its vertex between the Sun and Moon.

Taking as unity the Earth's equatorial radius, if p, p' are the

parallaxes, $\varkappa$, $\varkappa'$ the angular semi-diameters of the Moon and Sun respectively, then the distances are $\frac{1}{\sin p}$, $\frac{1}{\sin p'}$ and the radii are $\frac{\sin \varkappa}{\sin p}$, $\frac{\sin \varkappa'}{\sin p'}$ respectively; hence, if h, h' are the hour-angles west from Greenwich, Δ, Δ' the N.P.D.'s of the Moon and Sun respectively, we have

$$a = \frac{1}{\sin p} \sin \Delta \cos h, \quad a' = \frac{1}{\sin p'} \sin \Delta' \cos h',$$

$$b = \frac{1}{\sin p} \sin \Delta \sin h, \quad b' = \frac{1}{\sin p'} \sin \Delta' \sin h',$$

$$c = \frac{1}{\sin p} \cos \Delta \qquad , \quad c' = \frac{1}{\sin p'} \cos \Delta',$$

$$k = \frac{\sin \varkappa}{\sin p} \qquad , \quad k' = \frac{\sin \varkappa'}{\sin p'}.$$

And thence

$$\varrho = \frac{1}{\sin^2 p} (1 - \sin^2 \varkappa'),$$

$$\varrho' = \frac{1}{\sin^2 p'} (1 - \sin^2 \varkappa'),$$

$$\sigma = \frac{1}{\sin p \sin p'} [\cos \Delta \cos \Delta' + \sin \Delta \sin \Delta' \cos (h' - h) \mp \sin \varkappa \sin \varkappa'],$$

$$\mathrm{P} = \frac{1}{\sin p} \{\sin \Delta (x \cos h + y \sin h) + z \cos \Delta\},$$

$$\mathrm{P}' = \frac{1}{\sin p'} \{\sin \Delta' (x \cos h' + y \sin h') + z \cos \Delta'\},$$

Moreover, if the right ascensions of the Moon and Sun are α, α' respectively, and if the R.A. of the meridian of Greenwich (or sidereal time in angular measure) be $= \Sigma$, then we have

$$h = \Sigma - \alpha, \quad h' = \Sigma - \alpha'.$$

It is to be observed that $h - h'$, Δ, Δ' are slowly varying quantities, viz., their variation depends upon the variation of the celestial positions of the Sun and Moon; but h and h' depend on the diurnal motion, thus varying about 15° per hour; to put in evidence the rate of variation of the several angles h, h', Δ, Δ' during the continuance of the eclipse, instead of the foregoing values of h, h', I write

$$h' = \left\{ \mathrm{E} + \left(1 + \frac{\mathrm{E}_1 - \mathrm{E}}{24}\right) t \right\} 15^\circ,$$

there t is the Greenwich mean time, E, E_1 are the values (reckoned in parts of an hour) of the Equation of Time at the preceding and following mean noons respectively, taken positively or negatively, so that E, E_1 are the mean times of the two successive apparent noons respectively; whence also

$$h = \left\{ E + \left(1 + \frac{E_1 - E}{24} \right) t \right\} 15^\circ - \alpha + \alpha'.$$

And moreover

$$\begin{aligned} \alpha &= A + m\,(t - T), \\ \alpha' &= A + m'\,(t - T), \\ \Delta &= D + n\,(t - T), \\ \Delta' &= D' + n'\,(t - T), \end{aligned}$$

if T be the time of conjunction, A, A, D, D′ the values at that instant of the R.A.'s and N.P.D.'s; m, m' and n, n' the horary motions in R.A. and N.P.D. respectively.

It appears to me not impossible but that the foregoing form of equation

$$(x^2 + y^2 + z^2)(\varrho + \varrho' - 2\sigma) - (P - P')^2 - 2(\varrho' - \sigma)P - 2(\varrho - \sigma)P' + \varrho\varrho' - \sigma^2 = 0$$

for the umbral or penumbral cone might present some advantage in reference to the calculation of the phenomena of an eclipse over the Earth generally: but in order to obtain in the most simple manner the equation of the same cone referred to a set of principal axes, I proceed as follows:—

Writing

$$\begin{aligned} a &= b c' - b' c, & f &= a - a', \\ b &= c a' - c' a, & g &= b - b', \\ c &= a b' - a' b, & h &= c - c', \end{aligned}$$

(and therefore $a f + b g + c h = 0$)

Then, if

$$X = \frac{(b h - c g) x + (c f - a h) y + (a g - b f) z}{\sqrt{a^2 + b^2 + c^2}\ \sqrt{f^2 + g^2 + h^2}},$$

$$Y = \frac{a x + b y + c z}{\sqrt{a^2 + b^2 + c^2}},$$

$$Z = \frac{f x + g y + h z}{\sqrt{f^2 + g^2 + h^2}},$$

X, Y, Z, will be co-ordinates referring to a new set of rectangular axes; viz., the origin is, as before, at the centre of the Earth, the axis of Z is parallel to the line joining the centres of the Sun and Moon; the axis of X cuts at right angles the last-mentioned line; and the axis of Y is perpendicular to the plane

of the other two axes; or, what is the same thing, to the plane through the centres of the Earth, Sun, and Moon.

The co-ordinates of the vertex of the cone are therefore $X_\circ$, $Y_\circ$, $Z_\circ$, where these denote what the foregoing values of X, Y, Z, become on substituting therein for x, y, z, the values

$$\frac{k'a \mp k a'}{k' \mp k}, \qquad \frac{k'b \mp k b'}{k' \mp k}, \qquad \frac{k'c \mp k c'}{k' \mp k},$$

and the equation of the cone therefore is

$$(X - X_\circ)^2 + (Y - Y_\circ)^2 = \tan^2 \lambda \, (Z - Z_\circ)^2,$$

where

$$\sin \lambda = \frac{k' \mp k}{G}$$

if for a moment G denotes the distance between the centres of the Sun and Moon. We have therefore

$$\tan \lambda = \frac{k' \mp k}{\sqrt{G^2 - (k' \mp k)^2}},$$

or since

$$G^2 = (a' - a)^2 + (b' - b)^2 + (c' - c)^2,$$

this is in fact

$$\tan \lambda = \frac{k' \mp k}{\sqrt{\varrho + \varrho' - 2\sigma}},$$

where ϱ, ϱ', σ signify as before; and thus $X_\circ$, $Y_\circ$, $Z_\circ$, tan λ are all of them given functions of a, b, c, k, a', b', c', k', and consequently of the before-mentioned astronomical data of the problem. The form is substantially the same as Bessel's equation (3), *Ast. Nach.* No. 321 (1837), (but the direction of the axes of X, Y, is not identical with those of his x, y); and it is therefore unnecessary to consider here the application of it to the calculation of the eclipse for a given point on the Earth.

The Astronomische Gesellschaft.

The third biennial meeting was held at Vienna from the 13th to the 16th of September, 1869, under the presidency of M. Struve; the number of members present was 39; total number 216. The several subjects in question at the meeting of 1867 (*See Monthly Notices*, vol. xxviii. p. 268) were further discussed, and other subjects brought before the meeting; the principal ones were as follows:—

1. As to the New Tables of *Jupiter*, a large part of the

auxiliary calculations had been performed under the direction of Prof. Hansen, by junior members of the Society; and (his own part of the work being already accomplished) it was hoped that in the following year the definitive amendment of the elements could be taken in hand.

2. The reduction of the observations of the Periodic Comets had been proceeded with; a desideratum in reference thereto had been supplied by the completion of the *Tabulæ Quantitatum Besselianarum pro Annis* 1750 *ad* 1840, under the direction of MM. Struve and Auwers.

3. In reference to the new reduction of Bradley's observations, Prof. Auwers reported that the more mechanical part of the work was already accomplished. The number of zenith distances observed between 1750 and 1762 (exclusively of those of the Moon and planets, which were not in the first instance to be included in the new reduction) was about 19,000, of which 1650 related to fundamental stars.

4. Dr. Schmidt, Director of the Observatory at Athens, exhibited eight sheets of his new map of the Moon; the plan being that of Lohrman's, but the scale about double, the diameter being 6 Paris feet. It was stated that the publication of the map would hardly be completed within 10 years.

5. Plans were exhibited of the proposed New Observatory at Vienna.

6. In reference to the plan for the observation of the stars of the northern hemisphere up to the ninth magnitude, reports were received from MM. Struve, Kowalski, Krüger, Schwarz, Bruhns, Tiele, Auwers, and Safford, upon the progress which had been made: and a definitive programme was drawn up, which is published in the *Vierteljahrschrift*. It appears thereby that the work has been undertaken by the different observatories in zones as follows:—

80°	to	75°	Kasan.
75	,,	70	Dorpat.
70	,,	65	Christiania.
65	,,	55	Helsingfors.
55	,,	50	*Vacant.*
50	,,	40	Bonn.
40	,,	35	Chicago.
35°	,,	30°	Leipzig.
30	,,	25	Cambridge (E.)
25	,,	15	Berlin.
15	,,	10	Leipzig.
10	,,	4	Mannheim.
4	,,	1	Neuchatel.
+ 1	,,	− 2	Palermo.

7. Prof. Förster presented a report on the Eclipse-expedition

of 1868, which was intended to be published as a supplement to the *Vierteljahrschrift.*

8. Some discussion took place as to the proposed expedition for the observation of the transit of 1874.

The next meeting was fixed to be at Stuttgard, in 1871, under the presidency of M. Struve.

Besides the *Vierteljahrschrift* and the several works referred to *Monthly Notices*, vol. xxviii., there have been published by the Society.

Supplement-heft zu Jahrgang III.. von Asten Dr. E. *Neue Hulfstafeln zur Reduction der in der Histoire Céleste Française enthaltenen Beobachtungen*, 1868.

9. Lesser Dr. Otto. *Tafeln der Pomona mit Berücksichtigung der Störungen durch Jupiter, Saturn und Mars berechnet*, 1869.

Instrument for Sale.

A large spectroscope, recently constructed by Messrs. Simms for Mr. Huggins. The prisms are by Hofmann. It is adapted for use with the telescope, and suitable for observation of nebulæ, stars, and the Sun. It can be used for table purposes. To be seen at Mr. Ladd's, Beak Street, Regent Street, W.

CONTENTS.

	Page
Fellows elected	155
Notice as to the Eclipse of Dec. 22, 1870	ib.
On the Orbit of the Comet of 1683, by Mr. Plummer (Note by Mr. Hind)	156
Studies on the frequency of Sun-spots, and on their connexion with the Magnetic Declination-variation, by Prof. Wolf. (Translation) ..	157
Observations of Lunar Eclipse, Jan. 17, 1870, by Mr. Tebbutt ..	159
A few further Notes on the Floor of *Plato*, by Mr. Birt	160
On the Graphical Construction of the Umbral or Penumbral Curve at any instant during a Solar Eclipse, by Prof. Cayley	162
On the Geometrical theory of Solar Eclipses, by Prof. Cayley	166
The Astronomische Gesellschaft	168
Instrument for sale	170

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